FREE CONVECTION OF NONLINEARLY-VISCOUS LIQUIDS AROUND AXISYMMETRIC SOLIDS, ALLOWING FOR THE TEMPERATURE DEPENDENCE OF THE CONSISTENCY

Z. P. Shul'man, É. A. Zal'tsgendler, and V. I. Baikov

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The effect of the temperature dependence of the consistency coefficient, the transverse curvature of the solid, the parameter of non-Newtonian behavior, and the concentration factor on heat and mass transfer and friction in the free convection of nonlinearly-viscous liquids around axisymmetric solids is determined.

In the range of low shear velocities realized under conditions of free convection, many fluid media exhibit considerable anomalies of viscosity [1, 2]. Problems of free convection in non-Newtonian and especially nonlinearly-viscous liquids have therefore become particularly pressing and are of great interest in both theory and practice. Until recently the analysis of free-convection problems has been limited by the assumption of constant physical properties of the medium, for example, in [3, 4]. However, the temperature drops characteristic for conditions of free convection may be very considerable. Allowance for the temperature dependence of the rheological characteristics is then essential. This applies primarily to the consistency coefficient, i.e., an analog of viscosity. Free convection may be caused not only by a temperature field gradient but also by concentration inhomogeneity. It is therefore interesting to consider the simultaneous influence of these factors on friction and heat transfer during free convection.

The dimensionless equations of the spatial boundary layer for the free convection of a binary mixture of nonlinearly-viscous liquids around axisymmetric solids take the following form in the asymptotic approximation for a thin solid of revolution:

$$\frac{\partial}{\partial y}\left[\omega(\Theta_1) r\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right]+r[\Theta_1+K\Theta_2]=0, \tag{1}$$

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial y}(rv)=0, \tag{2}$$

$$u\frac{\partial\Theta_1}{\partial x}+v\frac{\partial\Theta_2}{\partial x}=\frac{1}{r}\frac{\partial}{\partial y}\left(r\frac{\partial\Theta_1}{\partial y}\right), \tag{3}$$

$$u\frac{\partial\Theta_2}{\partial x}+v\frac{\partial\Theta_2}{\partial y}=\frac{\mathrm{Pr}_1}{\mathrm{Pr}_2}\frac{1}{r}\frac{\partial}{\partial y}\left(r\frac{\partial\Theta_2}{\partial y}\right), \tag{4}$$

where $K=\mathrm{sign}(C_0-C_\infty)\left[\mathrm{sign}(C_0-C_\infty)\frac{\mathrm{Gr}_2}{\mathrm{Gr}_1}\right]^{\frac{1}{2-n}}$. The boundary conditions are

$$u=v=0,\ \Theta_1=\Theta_2=1 \text{ for } y=0,$$

$$\frac{\partial u}{\partial y}\to 0,\ \Theta_1\to 0,\ \Theta_2\to 0 \text{ for } y\to\infty. \tag{5}$$

The introduction of the dependent and independent variables and parameters

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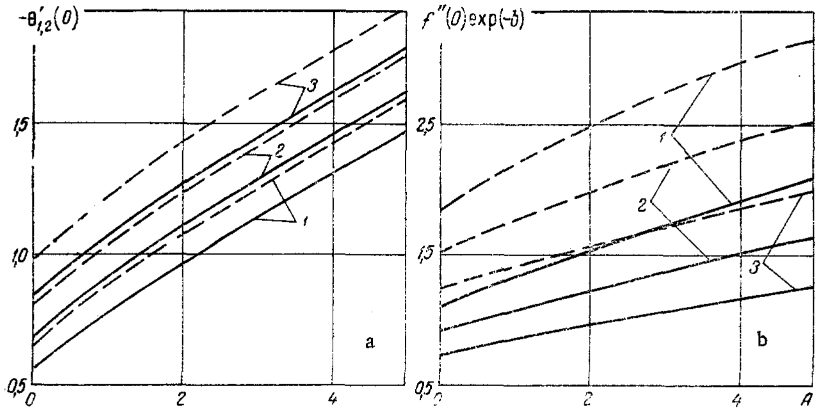

Fig. 1. Characteristics of heat transfer (a) and friction at the wall (b) as functions of the curvature parameter (n = 1): 1) b = 0; 2) 1; 3) 2. Continuous curve k = 0, broken curve k = 1.

$$\xi = \int_0^x U^\alpha(|x|)\, r_0^\beta(|x|)\, d|x|, \quad \eta = a\xi^{-\gamma} U(|x|) \int_0^y r dy,$$

$$\psi = b\xi^\gamma f(\eta), \quad A = \frac{2\xi^\gamma}{aU(|x|)\, r_0^2(|x|)}, \tag{6}$$

where ψ is determined from the continuity equation

$$ru = \frac{\partial\psi}{\partial y}, \quad rv = -\frac{\partial\psi}{\partial x},$$

and the requirement that the system of equations derived from (1)-(4) with due allowance for (6) should be automodel, leads to the equations

$$\alpha = -1, \quad \frac{1}{\gamma(\beta+2)} = \frac{n}{3n+1}, \; U = |x|^{\frac{n+1}{3n+1}}, \quad r_0 = c|x|^{\frac{n}{3n+1}}. \tag{7}$$

In order to eliminate the constants from the final system of equations we must put

$$a = c^{\beta\gamma-1}\gamma^\gamma, \quad b = c^{1-\beta\gamma}\gamma^{-\gamma}.$$

The problem then reduces to a system of nonlinear differential equations

$$\frac{d}{d\eta}\left[\omega(\Theta_1)(1+A\eta)^{\frac{n+1}{2}} |f''|^{n-1} f''\right] + \Theta_1 + K\Theta_2 = 0,$$

$$\frac{d}{d\eta}\left[(1+A\eta)\Theta_1'\right] + (\operatorname{sign} f') f\Theta_1' = 0, \tag{8}$$

$$\frac{d}{d\eta}\left[(1+A\eta)\Theta_2'\right] + \frac{\mathrm{Pr}_2}{\mathrm{Pr}_1}(\operatorname{sign} f') f\Theta_2' = 0,$$

with boundary conditions

$$f = f' = 0, \quad \Theta_1 = \Theta_2 = 1 \text{ for } \eta = 0,$$

$$f'' \to 0, \quad \Theta_1 \to 0, \quad \Theta_2 \to 0 \text{ for } \eta \to \infty. \tag{9}$$

The local heat- and mass-transfer coefficients and the coefficient of friction are determined from the equations

$$\mathrm{Nu}_{1,2} = -\Theta'_{1,2}(0)\,\mathrm{Gr}_1^{\frac{1}{2(n+1)}}\,\mathrm{Pr}_1^{\frac{n}{3n+1}}\,|x|^{\frac{2n+1}{3n+1}},$$

$$c_f = 2[f''(0)]^n \omega(1)\,\mathrm{Gr}_1^{-\frac{1}{2(n+1)}}\,\mathrm{Pr}_1^{-\frac{n+2}{3n+1}}\,|x|^{\frac{1}{3n+1}}. \tag{10}$$

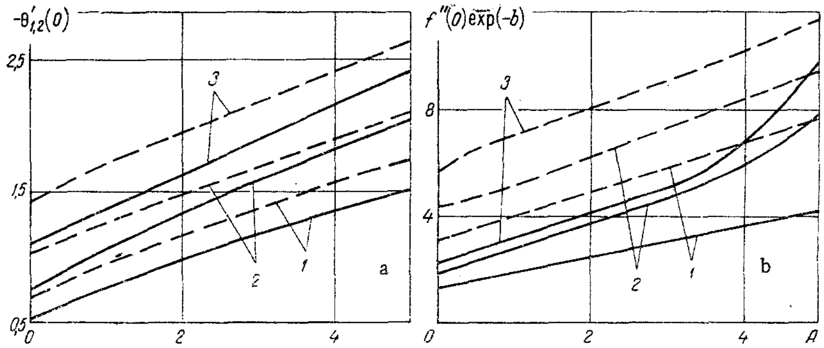

Fig. 2. Characteristics of heat transfer (a) and friction at the wall (b) as functions of the curvature parameter (n = 0.5): 1) b = 0; 2) 1; 3) 2. Continuous curve k = 0, broken curve k = 1.

The averaged coefficients respectively equal

$$\overline{\mathrm{Nu}}_{1,2} = -\frac{3n+1}{5n+2}\Theta'_{1,2}(0)\,\mathrm{Gr}_1^{\frac{1}{2(n+1)}}\mathrm{Pr}_1^{\frac{n}{3n+1}},$$

$$\bar{c}_f = \frac{2(3n+1)}{3n+2}[f''(0)]^n\omega(1)\,\mathrm{Gr}_1^{-\frac{1}{2(n+1)}}\mathrm{Pr}_1^{\frac{n+2}{3n+1}}. \qquad (11)$$

In order to obtain numerical results we must make the form of the $\omega(\Theta_1)$ relationship quite specific. In the present investigation we take an exponential temperature dependence of the consistency coefficient (the Reynolds relationship)

$$\omega(\Theta_1) = \exp(-b\Theta_1), \qquad (12)$$

which is fully justified for many real systems [5]. In certain papers, such as [6, 7], the index of the exponent in Eq. (12) includes a factor n. This does not introduce any major change into the calculation since the results of the present analysis may easily be converted to this kind of relationship.

The system of equations (8), with due allowance for Eq. (12), was solved numerically on the Minsk-22 computer by the modified Newton method for the case $\mathrm{Pr}_1 = \mathrm{Pr}_2$. Some of the results of the calculations are presented in Figs. 1 and 2. An increase in the parameter in Eq. (12) leads to an intensification of heat transfer (Figs. 1a and 2a). The effective viscosity falls, and this leads to a fall in the friction characteristics for the case of a Newtonian liquid (Fig. 1b) and to a rise in these characteristics as b increases for n = 0.5 (Fig. 2b). Thus, allowance for the temperature dependence of the consistency coefficient leads to a qualitatively new picture of the whole process.

Changes in the curvature parameter A and also an increase in the value of K act in qualitatively the same manner for all n; with increasing A or K both the heat and mass transfer and the friction on the surface of the solid increase (Figs. 1 and 2). When the thickness of the boundary layer is much smaller than the radius of the solid of revolution ($\delta/r_0 \ll 1$) the spatially axisymmetric problem is reduced to the system of equations (8) (case A = 0) by the introduction of the variables

$$\eta = y\left[\frac{3n+1}{2n+1}\left(\frac{1}{r_0^n\cos\alpha_1}\right)^{\frac{3n+1}{n(2n+1)}}\int_0^X(\cos\alpha_1 r_0^n)^{\frac{1}{2n+1}}dX\right]^{-\frac{n}{3n+1}},$$

$$u = (\cos\alpha_1)^{\frac{1}{n}}\left[\frac{3n+1}{2n+1}\left(\frac{1}{r_0^n\cos\alpha_1}\right)^{\frac{3n+1}{n(2n+1)}}\int_0^X(\cos\alpha_1 r_0^n)^{\frac{1}{2n+1}}dX\right]^{\frac{n(n+1)}{3n+1}},$$

$$X = \int_0^{|x|} r_0(|x|)\,d|x|.$$

Here the heat- and mass-transfer coefficients are respectively written

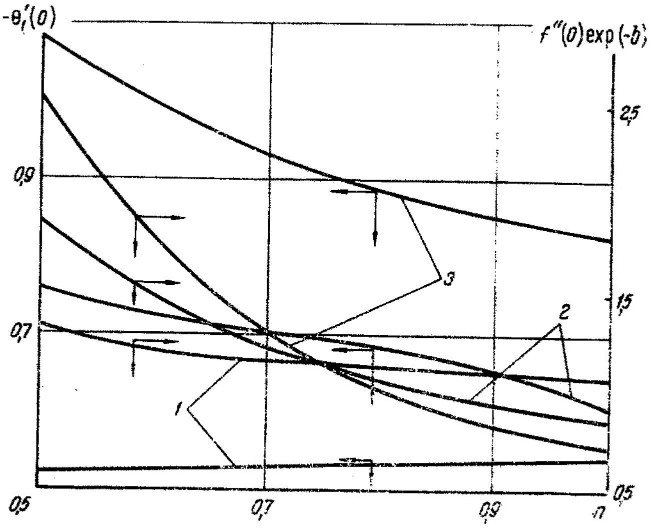

Fig. 3. Characteristics of heat transfer and friction at the wall as functions of the parameter of non-Newtonian behavior (A = 0, k = 0): 1) b = 0; 2) 1; 3) 2.

$$\mathrm{Nu}_{1,2} = -\Theta'_{1,2}(0)\left(\frac{2n+1}{3n+1}\right)^{\frac{n}{3n+1}}\mathrm{Gr}_1^{\frac{1}{2(n+1)}}\mathrm{Pr}_1^{\frac{n}{3n+1}}\frac{r_0^n\cos\alpha_1}{\left[\int\limits_0^{|x|} r_0^{\frac{3n+1}{2n+1}}(\cos\alpha_1)^{\frac{1}{2n+1}}d|x|\right]^{\frac{n}{3n+1}}}.$$

In particular cases we have:

1. Sphere $\cos\alpha_1 = \sin$, $r_0 = \sin x$.
2. Cone $\cos\alpha_1 = \mathrm{const}$, $r_0 = x\sin\alpha_1$.
3. Spatial critical point $\cos\alpha_1 = x$, $r_0 = x$.
4. Vertical cylinder $\cos\alpha_1 = 1$, $r_0 = 1$.

The system of equations (8) (case A = 0) also constitutes the reduced form of problems of the free convection of a binary mixture of non-Newtonian liquid around flat solids, in particular free convection around 1) a vertical plate, 2) a horizontal cylinder, 3) a plane critical point, 4) a wedge. In this case we must introduce the new coordinates proposed in [3], replacing x by $|x|$ in these.

The results of our numerical calculations of the problem of free convection around these eight types of solid are shown in Fig. 3. We note that, whereas the heat-transfer coefficient increases monotically with increasing values of the parameter b for the n values studied, the friction characteristics increase with rising b for extremely pseudoplastic liquids ($n \lesssim 0.75$) and diminish as the pseudoplasticity becomes weaker ($n > 0.75$).

NOTATION

$x = \frac{x'}{L}$, $y = \frac{y'}{L}\mathrm{Gr}_1^{\frac{1}{2(n+1)}}\mathrm{Pr}_1^{\frac{n}{3n+1}}$, $r = \frac{r_0' + y'}{L}\mathrm{Gr}_1^{\frac{1}{2(n+1)}}\mathrm{Pr}_1^{\frac{n}{3n+1}}$, dimensionless coordinates; x', y', dimensional coordinates; L, characteristic size; $\Theta_1 = \frac{T - T_\infty}{T_0 - T_\infty}$, dimensionless temperature; $\Theta_2 = \frac{C - C_\infty}{C_0 - C_\infty}$, dimensionless concentration; T_0, C_0, absolute temperature and concentration at the wall; T_∞, C_∞, absolute temperature and concentration as $y \to \infty$; $\mathrm{Pr}_1 = \frac{\rho c_p}{\lambda}\left(\frac{k}{\rho}\right)^{\frac{2}{1+n}} L^{\frac{1-n}{1+n}}[L\beta g(T_0 - T_\infty)]^{\frac{3(n-1)}{2(n+1)}}$, $\mathrm{Pr}_2 = \frac{\rho c_p}{D}\left(\frac{k}{\rho}\right)^{\frac{2}{1+n}} L^{\frac{1-n}{1+n}}[L\beta g(C_0 - C_\infty)]^{\frac{3(n-1)}{2(n+1)}}$, modified Prandtl numbers; $\mathrm{Gr}_1 = \frac{\rho^2 L^{n+2}[\beta g(T_0 - T_\infty)]^{2-n}}{k^2}$, $\mathrm{Gr}_2 = \frac{\rho^2 L^{n+2}[\beta g(C_0 - C_\infty)]^{2-n}}{k^2}$, modified Grashof numbers; λ, thermal conductivity; β, volume expansion coefficient; g, gravitation acceleration; k, consistency coefficient; n, parameter of non-Newtonian behavior; ψ, modified stream function; $\omega(\Theta_1)$, function representing the temperature dependence of the consistency; r_0, radius of the solid of revolution; η, $f(\eta)$, automodel variables; A, curvature parameter; $Nu_{1,2}$, c_f, local Nusselt numbers and coefficient of friction; $\overline{Nu}_{1,2}$, $\overline{c}_f$ averaged Nusselt numbers and coefficient of friction; α_1, angle between the normal to the contour of the solid and the direction of gravity.

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